



SECOND ORDER SLIP EFFECT ON MICROPOLAR FLUID PAST A STRETCHING SHEET

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ABSTRACT

This paper is to look at the impact of second order slip flow on MHD boundary layer flow of micropolar fluid past a stretching sheet. The administering non-direct conditions were decreased into coupled non-straight higher order standard differential condition by utilizing fitting comparably changes. At that point numerical methodology has been dealt with to illuminate the differential conditions. We have utilized Runge-Kutta fourth order strategy with shooting procedure to fathom the ODEs and utilized MATLAB programming to drawing diagrams of velocity, temperature and microrotation profiles for various benefits of administering parameters. Likewise varieties of the skin grating coefficient and nearby Nusselt number are demonstrated graphically with the end goal to watch the physical unsettling influences in the profiles. The outcomes show that the skin friction coefficient c_f increases as the estimations of slip parameter γ increase. However, the local Nusselt number $-\theta'(0)$ decreases as both slip parameters γ and δ increase. Excellent agreement observed with previous study.

Keywords: *Second order slip flow, Micropolar, MHD, Heat transfer, Stretching sheet*

INTRODUCTION

Numerous specialists demonstrated their enthusiasm for research papers which are managed the MHD boundary layer flow of a micropolar fluid past a stretching sheet because of the way that the boundary layer flow and heat transfer of a micropolar fluid has a huge significance in designing applications. For instance, it is essential in the field of synthetic preparing materials which are valuable in polymeric fluids, foodstuffs and slurries. The spearheading take a shot at micropolar fluid was done first by Eringen [1, 2] about 50 years

prior. By utilizing this hypothesis, the scientific model of numerous non-Newtonian fluids was produced for which the established Navier–Stokes hypothesis is improper. From that point forward, the investigation of a micropolar fluid has the consideration of numerous analysts in the regions of fluid science and building inferable from its huge applications in numerous cutting edge

Nomenclature		Greek Symbols	
a	stretching constant	α_1	momentum accommodation coefficient
A, B	slip constants	β	material parameter
B_0	magnetic field strength	γ	first order slip parameter
C_f	skin friction coefficient	δ	second order slip parameter
c_p	specific heat	η	dimensionless similarity variable
f	dimensionless flow function	θ	dimensionless temprature
g	dimensionless microrotation function	μ	coefficient of dynamic viscosity
j	micro-inertia density	ν	coefficient kinematic viscosity
k	thermal conductivity of fluid	σ	electrical conductivity
K_n	Knudsen number	ψ	flow function
M	magnetic parameter	λ	molecular mean free path
N	microrotation component	α	thermal diffusivity
n	microrotation parameter	ρ	fluid density
Nu_x	local Nusselt number	κ	coefficient of vortex viscosity
P_r	Prandtl number	Ω	spin-gradient viscosity
q_w	surface heat flux	τ_w	wall shear stress
Re_x	local Reynolds number	Subscripts	
T	temperature of the fluid	∞	condition at the free flow
T_w	temperature at the surface	w	condition at the surface
T_∞	ambient temperature	u, v	velocity component along x – and y – direction

Numerous researchers have distributed papers on micropolar fluid by thinking about various angles under various circumstances. Especially, the investigation of boundary layer flow of micropolar fluid past a stretching surface has gotten more consideration than other geometrical surfaces. This is on the grounds that the boundary layer flow of an incompressible micropolar over a stretching sheet has vital applications in numerous modern and building procedures, for example, in polymer businesses. Particularly, the investigation of flow of micropolar fluids because of stretching sheet has central significance in numerous modern and building applications, for example, fluid precious stone, weaken arrangement of polymers and suspensions and so on [3]. In any case, the investigation of micropolar over



shrinking sheet is another angle. Appropriately, Yacob and Ishak [4] numerically analyzed the boundary layer flow of a micropolar fluid past a shrinking sheet. Also Ishak [5] contemplated the impact of thermal radiation on the boundary layer flow of micropolar fluid past a stretching sheet. It was shown that radiation decreases the heat transfer rate at the surface. Besides, Ishak et al. [6– 8] inspected MHD stagnation point flow of a micropolar fluid towards a stretching, vertical and wedge. The examination of convection in a doubly stratified micropolar fluid was broadly considered by Sinivasacharia and RamReddy [9– 11]. Their numerical outcome showed that the estimations of microrotation changes in sign with in the boundary layer. The investigation of slip flow is a traditional issue, be that as it may, it is as yet a functioning examination zone. Numerous researchers have analyzed slip flow under various viewpoints. For example, Anderson [12, 13] inspected the impacts of slip, gooey dissemination, joule thermaling on boundary layer flow of a stretching surface. In addition, researchers in Refs. [14– 18] stretched out the examination to a non-Newtonian fluid and broke down the impacts of incomplete slip, thick dissemination, joule thermaling and MHD pasta stretching surface. Besides, Das [19] examined the fractional slip flow on MHD stagnation point flow of micropolar fluid past a shrinking sheet. The outcomes demonstrate that an expansion in slip parameter diminishes the microrotation profile diagrams. Even more, Noghrehabadi et al. [20] broadened the investigation of slip flow to a nanofluid and numerically analyzed the impact of halfway slip boundary condition on the flow and heat transfer of nanofluid past stretching sheet with endorsed steady divider temperature. Besides, Wubshet and Shanker [21] stretched out velocity slip boundary condition to thermal and solutal slip flow to a nanofluid and numerically examined MHD boundary layer flow and heat transfer of a nanofluid past a porous stretching sheet with velocity, thermal and solutal slip boundary conditions. The previously mentioned investigations consider just the 1st order slip flow. Be that as it may, second order slip flow happens in numerous mechanical zones, however analysts have not given careful consideration on it. Thusly, this examination has given a full thought to inspect the impact of second order slip flow. As needs be, Fang et al. [22] and Fang and Aziz [23] talked about gooey flow of a Newtonian fluid over a shrinking sheet with second order slip flow demonstrate. Essentially, Mahantesh et al. [24] analysed second order slip flow and heat transfer over a stretching sheet with non-direct boundary condition. They demonstrated that both the first and the second order slip parameter fundamentally influence the shear pressure. Besides, Rosca and Pop [25, 26] researched the second order slip flow and heat transfer over a vertical penetrable stretching/shrinking sheet. The aftereffect of their examination demonstrated that flow and heat transfer qualities are firmly impacted by blended convection, mass transfer and slip flow display parameters. By thinking about attractive field, Turkyilmazoglu [27] considered systematically heat and mass transfer of MHD second order slip flow over a stretching sheet. Their investigation demonstrates that expanding the estimations of attractive parameter and second order slip parameter extensively decrease the

size of shear worry at the divider. Sing and Chamkha [28] additionally examined the double answer for thick fluid flow and heat transfer with second-order slip at linearly shrinking isothermal sheet in a peaceful medium. Pattnaik et al. [29-36] concentrated the conduct of MHD fluid flow and watched some intriguing outcomes.

According to author's knowledge, no examinations has been accounted for which talks about the impact of second order slip boundary condition on MHD boundary layer flow and heat transfer of micropolar fluid over an stretching sheet. Along these lines, this investigation is focused to fill this information hole. This examination look at the impact of first, second order slip flow and attractive parameter on boundary layer flow towards stretching sheet in micropolar fluid. The administering boundary layer conditions were changed into a two-point boundary esteem issue utilizing likeness factors and numerically unravelled utilizing bvp4c from MATLAB. The impacts of physical parameters on fluid velocity, temperature and microrotation were talked about and appeared in diagrams.

Mathematical formulation

We have considered a steady laminar boundary layer flow of a micropolar fluid past a stretching sheet with second order slip boundary condition with a constant temperature T_w . The uniform ambient temperature is given by T_∞ . It is assumed that the sheet is stretched with a velocity $u_w = ax$. The flow is subjected to a constant transverse magnetic field of strength B_0 which is assumed to be applied in the positive y-direction, normal to the surface. The coordinate frame selected in such a way that x-axis is stretching along the stretching sheet and y-axis is normal to it. The corresponding governing equations are as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu + \kappa}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\Omega \gamma}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j} \left(2N + \frac{\partial u}{\partial y} \right) \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (4)$$

The boundary conditions are:

$$\begin{aligned} u = u_w + U_{slip}, v = 0, N = -n \frac{\partial u}{\partial y}, T = T_w, \text{ at } y = 0 \\ u \rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{ as } y \rightarrow \infty \end{aligned} \quad (5)$$

where $\Omega = \mu \left(1 + \frac{\beta}{2} \right)$.

The slip velocity at the surface is given by:

$$U_{slip} = A \frac{\partial u}{\partial y} + B \frac{\partial^2 u}{\partial y^2} \tag{6}$$

where $A = \frac{2}{3} \left(\frac{3 - \alpha_1 l^2}{\alpha_1} - \frac{3}{2} \frac{1 - l^2}{K_n} \right)$, $B = -\frac{1}{4} \left(l^4 + \frac{2}{K_n^2} (1 - l^2) \right)$, $l = \min \left[1, \frac{1}{K_n} \right]$, $0 \leq \alpha_1 \leq 1$ are

constant. Based on the construction of l and for given value of K_n , we have $0 \leq l \leq 1$. The molecular mean free path is always positive. Thus, it is known that $B < 0$ and hence the second term in the right hand side of Eq. (6) is a positive number.

Using the similarity variable and dimensionless functions as:

$$\eta = \sqrt{\frac{a}{\nu}} y, f(\eta) = \frac{\psi}{x\sqrt{av}}, \omega(\eta) = \frac{N}{ax\sqrt{\frac{a}{\nu}}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

The equation of continuity is satisfied for:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

and the governing Eqs. (1)– (4) are reduced to:

$$(1 + \beta) f''' + f f'' + \beta g' - (f')^2 - M f' = 0 \tag{7}$$

$$\left(1 + \frac{\beta}{2} \right) g'' - \beta (2g + f'') + f g' - f' g = 0 \tag{8}$$

$$\theta'' + P_r f \theta' = 0 \tag{9}$$

with boundary conditions

$$\text{at } \eta = 0: f(0) = 0, g(0) = -n f''(0), \theta(0) = 1, f'(0) = 1 + \gamma f''(0) + \delta f'''(0)$$

$$\text{as } \eta \rightarrow \infty: f' \rightarrow 0, g \rightarrow 0, \theta \rightarrow 0 \tag{10}$$

$$\text{where } M = \frac{\sigma B_0^2}{a\rho}, P_r = \frac{\nu}{\alpha}, \gamma = A\sqrt{\frac{a}{\nu}}, \delta = B\frac{a}{\nu}, \beta = \frac{\kappa}{\mu}.$$

Physical quantities

The physical quantities of engineering interest are the skin friction coefficient C_f and local Nusselt number Nu_x which are defined as:

$$C_f = \frac{\tau_w}{\rho u_w^2}, Nu = \frac{xq_w}{k(T_w - T_\infty)} \tag{11}$$

The wall shear stress and heat transfer from the plate, respectively, are given by,

$$\tau_w = \left[(\mu + k) \frac{\partial u}{\partial y} + k\omega \right]_{y=0} \quad \text{and} \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

So from equation (11) we get,

$$C_f \sqrt{\text{Re}_x} = -(1 + \beta(1-n))f''(0), \quad Nu / \sqrt{\text{Re}_x} = -\theta'(0) \quad (12)$$

$$\text{where } \text{Re}_x = \frac{ax^2}{\nu} .$$

Results and discussion

The coupled three ordinary differential Eqs. (7)–(9) with the boundary conditions Eq. (10) are solved numerically by using Runge-Kutta 4th order method with shooting technique. Variations in non-dimensional velocity profile for different values of pertinent parameters were shown in Fig. 2 (a-d). Velocity profile is decreased for increasing values of first order slip parameter (γ) but reverse effect has been observed for increasing values of second order slip parameter (δ), which can be checked in cases (a) and (c) respectively. In case (b), it has been noticed that for increasing values of material parameter (β), velocity profile gets accelerated but in case (d), due to the resistive force offered by Lorentz force, motion of the fluid gets decelerated for increasing values of magnetic parameter (M). In Fig. 3(a-d), variation of microrotation profile for different values of pertinent parameters were shown. It has been observed that same reading occurred in cases of (a-c) as in case of velocity but in case (d) microrotation profile increased near the boundary but gets decreased thereafter for increasing values of magnetic parameter (M). Variations in non-dimensional temperature profile for different values of pertinent parameters were shown in Fig. 4 (a-d). Temperature profile is increased for increasing values of both first order slip parameter (γ) and second order slip parameter (δ), but reverse effect has been observed for increasing values of material parameter (β). But since velocity profile gets decelerated in case (d), due to Lorentz force, the fluid temperature gets accelerated for increasing values of magnetic parameter (M). Skin friction coefficient (C_f) gets accelerated for increasing values of first order slip parameter (γ), second order slip parameter (δ) and also for material parameter (β) but reverse effect has been occurred for increasing values of magnetic parameter (M) which can be observed in Fig. 5 (a-d). Fig. 6(a-d) is the evidence of variations in Nusselt number (Nu_x). The local Nusselt number decreases for increased values of both first order slip parameter (γ) and magnetic parameter (M) but it increases for increased values of both second order slip parameter (δ) and material parameter (β). As the physical point of view, we have considered the variations of velocity (f'), microrotation (g) profiles, Skin

friction coefficient (C_f) and Nusselt number (Nu_x) with increasing values of microrotation parameter (n). It has been observed that velocity and skin friction coefficient decreases but microrotation profile increases whereas Nusselt number decreases near the boundary but increases thereafter.

CONCLUSIONS

In this paper, the effects of second order slip flow and magnetic field on boundary layer flow and heat transfer of micropolar fluid over a stretching sheet were discussed. From the study, it is found that the flow velocity and the skin friction coefficient are strongly influenced by slip and material parameters. It is also observed that the velocity boundary layer thickness decreases as the absolute values of slip parameters increase. However, the thermal boundary layer thickness increases as the absolute values of the two slip parameters increase. Furthermore, the skin friction coefficient $-f''(0)$ and the local Nusselt number $-\theta'(0)$ decrease as the absolute values of slip parameters increase.

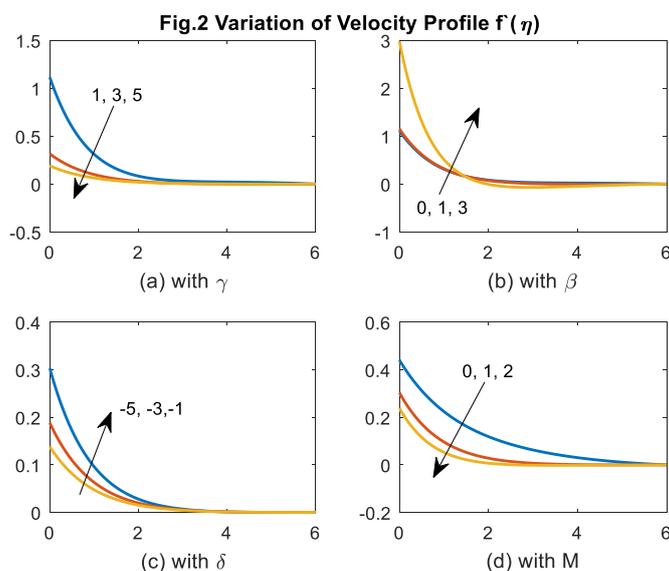


Fig.3 Variation of Microrotation Profile $g(\eta)$

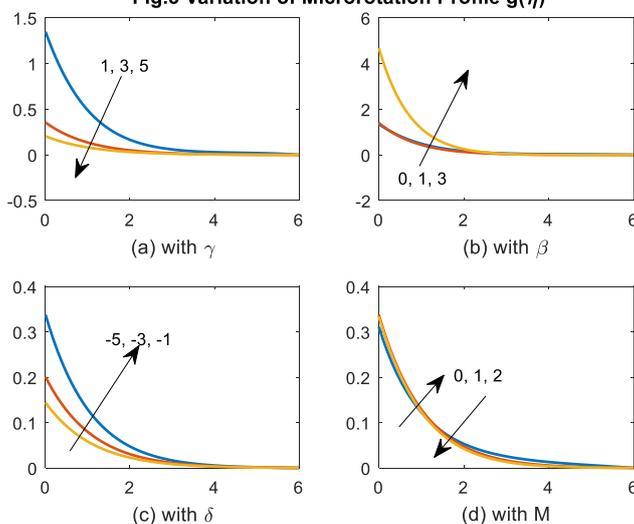


Fig.4 Variation of Temperature Profile $\theta(\eta)$

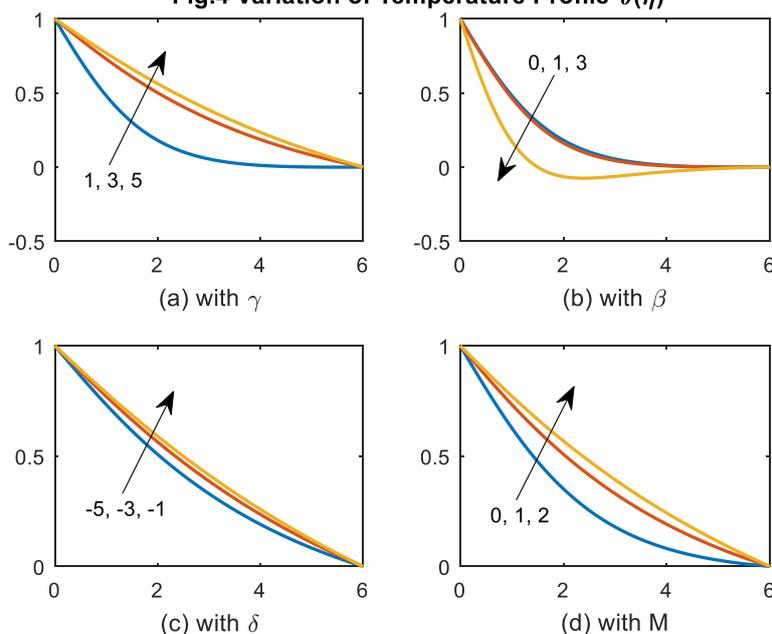


Fig.5 Variation of Skin friction coefficient C_f

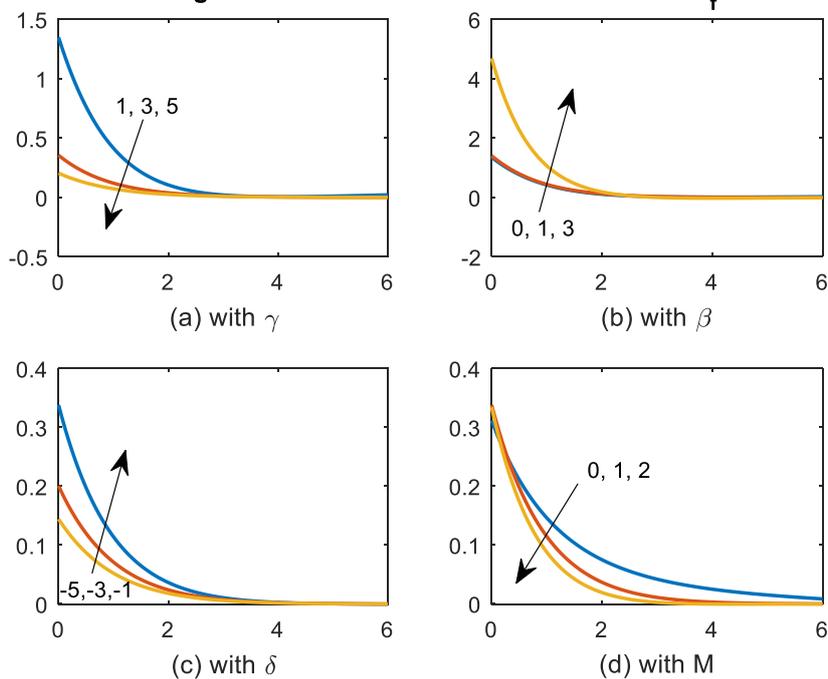


Fig.6 Variation of Nusselt Number Nu_x

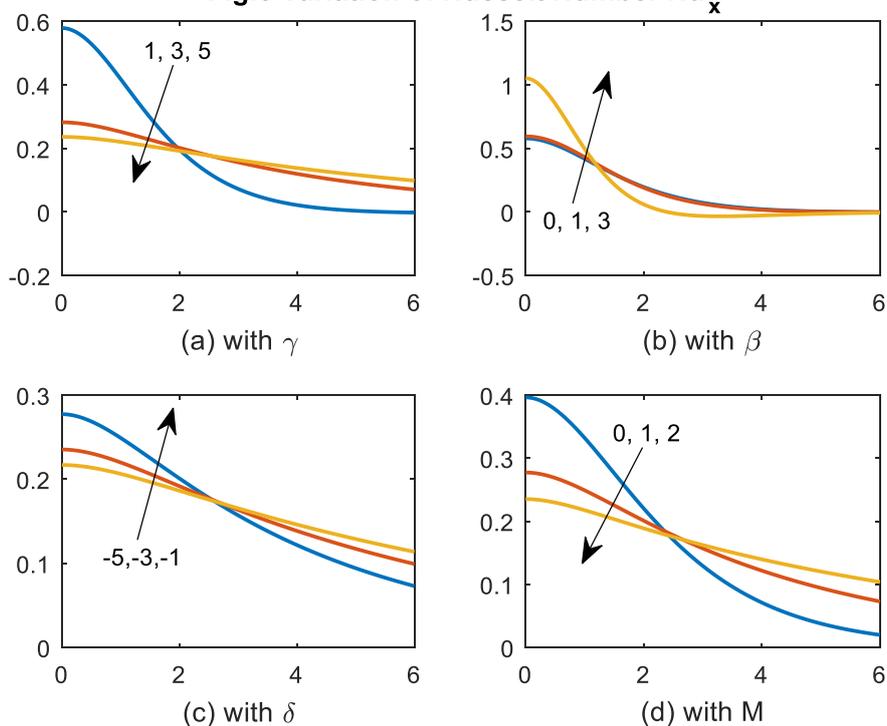
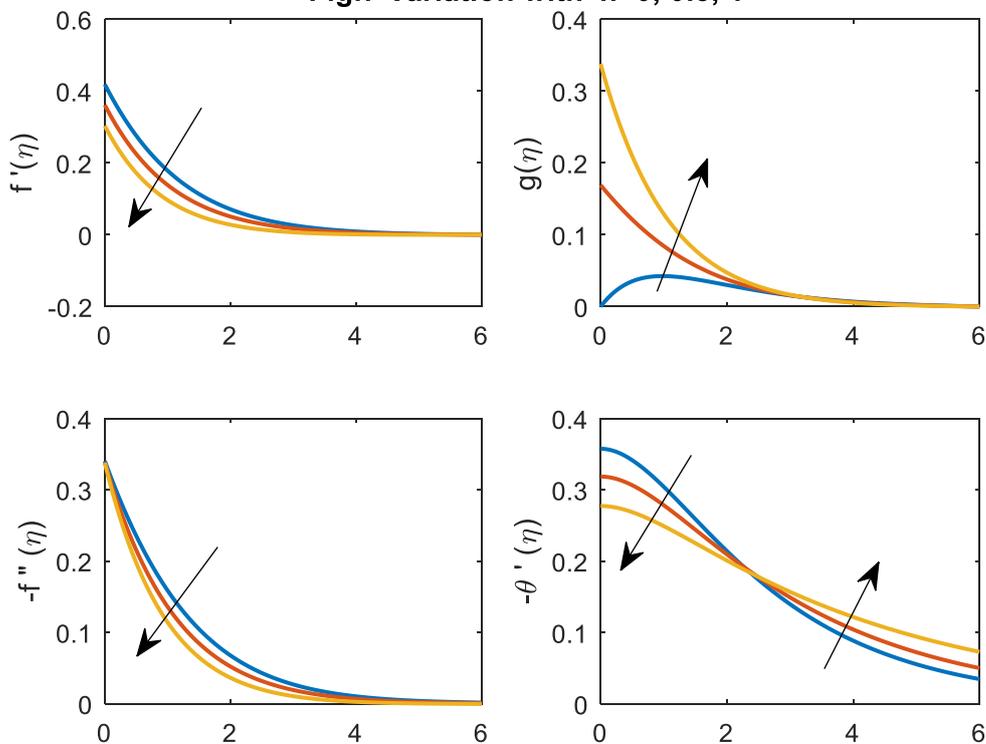


Fig.7 Variation with 'n=0, 0.5, 1'



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